More on the onset of core collapse (type II SN)

call that from H burning ($T_{ignition} \simeq 2\times10^7$ K) successive burning in the cores of massive stars proceeds through ^4He, ^{20}Ne, ^{16}O, and ^{28}Si ($T_{ign} \simeq 3\times10^9$ K) terminating at ^{56}Fe (nucleus with maximum binding energy per nucleon). Most all 1-D calcs show that the cores reach mass $M_c \simeq 1.5 M_\odot$ for initial stellar masses $8M_\odot \lesssim M \lesssim 20 M_\odot$.

Arnett (1979) offered a nice explanation for the tendency of all the models to converge to this final state:

In the late stages of evolution, core is convectively unstable, and thus thoroughly mixed implying core is relatively uniform. Now consider the equation of state, which is dominated by electrons:

(kinetic pressure term) (degenerate pressure term)

$$\frac{P}{\rho} \simeq \frac{Y_e k T}{m_B} + \bar{K}_\Gamma Y_e^\Gamma \rho^{\Gamma-1} \qquad (19)$$

$m_B \simeq m_p$

$Y_e = \frac{\#\text{ electrons}}{\text{baryon}} = \frac{n_e}{n_B} = \frac{1}{\mu_e}$

$\bar{K}_\Gamma$ = constant that depends on polytropic index Γ, which varies from $\frac{4}{3}$ (relativistic) $\leq \Gamma \leq 5/3$ (non relativistic)

For spherical configuration of mass M and radius R, equation of hydrostatic equilibrium ($\rho \nabla \Phi = -\nabla P$) implies roughly

$$\frac{P_c}{\rho_c} \simeq \frac{GM}{R} \simeq f\, G M^{2/3} \rho_c^{1/3} \qquad (20)$$

where "c" indicates central values and $f = f(\Gamma)$ is a numerical factor.

Using $P/\rho = P_c/\rho_c$ in (19) and combining with (20) gives:

↙ central temp

$$\frac{Y_e k T_c}{m_B} = f\, G M^{2/3} \rho_c^{1/3} - K_\Gamma Y_e^\Gamma \rho_c^{\Gamma-1} \qquad (21)$$

Now consider the maximum T_c a given configuration can achieve. For large M, first term on right of (21) dominates and then $T_c \propto \rho_c^{1/3}$. Thus continued contraction would lead to higher temps, and "any" fuel will eventually ignite. However, for small M and $\Gamma > \frac{4}{3}$, $T_c \to 0$ from (21) when

$$\rho \Rightarrow \rho_{crit} = \left(\frac{f G M^{2/3}}{K_\Gamma Y_e^\Gamma} \right)^{\frac{1}{(\Gamma - 4/3)}} \qquad (22)$$

So, when $\rho < \rho_{crit}$, we have $T_c \propto \rho_c^{1/3} M^{2/3}$ and when $\rho \to \rho_{crit}$ we have $T_c \to 0$, and $\rho > \rho_{crit}$ is unphysical.

This implies that the core actually passes through a maximum T_c during contraction, and then cools as it approaches the final state when degenerate electrons dominate the pressure.

Roughly, the mass marking the transition between the $T_c \propto \rho_c^{1/3} M^{2/3}$ dependence and $T_c \to 0$ can be estimated by the Chandrasekhar mass, for which $T_c = 0$, $\Gamma = 4/3$. Using (21), we then have:

$$\Rightarrow M_{crit} = \underbrace{\left(\frac{K_{4/3}}{fG}\right)^{3/2}}_{5.8} Y_e^2 \simeq 5.8\, Y_e^2 M_\odot , \qquad (23)$$

$5.8 \Leftarrow$ for $n=3$ polytrope where $p \propto \rho^{1+1/n}$

Silicon burning, which is the final core burning stage requires very high temps $kT \gtrsim 0.6 m_e c^2$, and thus one expects the cores to approach the limiting mass near the onset of Si burning. If the core mass $M_c < M_{crit}$, then system must wait until more ash from pre-Si burning phase is dumped into core for Si burning to start. Thus condition for Si burning go to completion →

in core is $M_{core} \gtrsim M_{si,ig} \simeq M_{si,ig} \simeq 1.2 M_\odot$

(Note that very low mass stars never get there)..

However: note also that if M_{core} were much larger than $M_{si,ig}$ at some earlier phase of core evolution, then contraction would rapidly proceed to high central temperatures and between burning stages neutrino energy release would reduce the entropy gradient, reducing convection, and thus reducing mixing, resulting in the now central core being a smaller region of the initial core, and a smaller mass. In this way, the system evolves so that the core is driven toward a mass M_{crit} prior to Si burning. (confirmed by simulations)

The ash from Si burning is primarily ^{56}Fe, ^{58}Fe, ^{60}Fe, ^{62}Ni)..

→

In short, an $8M_\odot < M < 20M_\odot$ will have a core mass just slightly larger than the M_{crit} of $1.2\ M_\odot$ (typically simulations show $M_{core} \simeq 1.5\ M_\odot$) before the core becomes neutron degenerate and collapse ensues. More specifically, just before the onset of collapse:

$T_c \simeq 8 \times 10^9 K$, $\rho_c \simeq 3.7 \times 10^9 g/cm^3$
$Y_{e_i} \simeq 0.42$, $M_{core} \simeq 1.5\ M_\odot$. The dissociation of Fe and neutronization of the core which subsequently ensue are endothermic, as discussed above, and lead to collapse.

Neutrino luminosity

As the opacity increases during core collapse, the neutrinos find escape more and more difficult. At densities above

$$\rho_{trap} = 3 \times 10^{11} g/cm^3 \qquad (24)$$

neutrinos are trapped; they co-move with the matter and actually build up a degenerate sea. At $\rho \simeq \rho_{trap}$, time scale for neutrinos to diffuse from core becomes comparable to collapse time. If we use this to define ρ_{trap} we can see how it is derived:

Hydrodynamical collapse time scale is of order the free-fall time scale:

$$t_{col} \simeq \frac{1}{(G\rho)^{1/2}} \simeq 4 \times 10^{-3} \rho_{12}^{-1/2} \text{ sec} \quad (25)$$

$\uparrow \rho_{12} \equiv \left(\frac{\rho}{10^{12}\,g/cm^3}\right)$

where $\rho = \frac{4}{3}\pi R^3 M_c$ is the mean density of the collapsing core, with $M_c \simeq 1.5\,M_\odot$ as above. The neutrino diffusion time scale can be estimated by assuming that coherent scattering is the dominant opacity source:

$$t_{diff} \simeq \frac{\lambda_A^{coh} N_{sc}}{c} \quad (26)$$

where λ_A^{coh} is the mean free path of a neutrino amidst heavy nuclei from coherent scattering; $N_{sc} \gg 1$ is # of scatterings before escape.

(e.g. $\nu + (Z,A) \to \nu + (Z,A)$)
↑ coherent scattering

[note that other neutrino absorption & scattering processes include free nucleon scattering: $\nu + n \to \nu + n$
$\nu + p \to \nu + p$
nucleon absorption $\nu_e + n \to p + e$
$e^- - \nu$ scattering $e^- + \nu \to e^- + \nu$]

Coherent scattering induces a random walk trajectory for the neutrino, but without changing its energy. We then have, for a random walk

$$\lambda_A^{coh} N_{sc}^{1/2} \sim R \text{ or } N_{\nu g_b} \qquad (27)$$

which is the condition for escape. (similar to photons incurring elastic Thomson scattering). The mean free path can be estimated from the cross section for coherent scattering σ_A:

$$\frac{1}{\lambda_A^{coh}} = n_A \sigma_A^{coh} = \left(\frac{\rho}{A m_B}\right) \sigma_A^{coh} \qquad (28)$$

nuclear physics calcs give
$\sigma_A^{coh} = \sigma_A^{coh}(E_\nu) \simeq 10^{-45} \left(\frac{E_\nu}{m_e c^2}\right)^2 A^2 \text{ cm}^2$ for characteristic $E_\nu \simeq 33 \rho_{12}^{1/3}$ MeV for electron capture onto proton $(e + p \to n + \nu)$]

then correct numerical constants in σ_A^{coh} are included for $A \sim 56$:

$$\left(\lambda_A^{coh}\right)^{-1} = 4 \times 10^{-5} \text{ cm}^{-1} \rho_{12}^{5/3} \qquad (29)$$

Using (24), (26) and (27), we have:

$$t_{diff} = \frac{\lambda_A^{coh} N_{sc}}{c} \simeq \frac{R^2}{\lambda_A^{coh} c} = \frac{R^2}{c} 4\times10^{-5} \rho_{12}^{5/3} \text{ sec}$$

since $R^2 = \left(\frac{M}{4/3\pi\rho}\right)^{2/3} \Rightarrow$

$$t_{diff} = \frac{(2\times10^{33})^{2/3}}{(4/3\pi)^{2/3}(10^{12})^{2/3}} \left(\frac{4\times10^{-5}}{c}\right)\left(\frac{M}{M_\odot}\right)^{2/3} \rho_{12}$$

$$\simeq 0.1\, \rho_{12} \text{ sec} \qquad (30)$$

Eqn (30) and (25) show that

$$t_{dif} \simeq t_{ff} \text{ when} \qquad (31)$$

$$\rho = \rho_{trap} \simeq 1.4\times10^{11} \text{ g/cm}^3$$

Thus, as advertised we have derived the critical density for neutrino trapping!

This result (31) is important because neutrino luminosities are greatly reduced from trapping: Once center of core reaches 3×10^{14} g/cm^3, neutron degeneracy prevents further collapse. and most of the core luminosity is emitted via neutrinos.

In the absence of trapping, the total binding energy released in the core (which is most of its gravitational energy) would be released on a collapse time scale (e.g. a free fall time at $R_{nuc} \simeq 12$ km for $M \simeq M_\odot$)

This would produce a neutrino luminosity of

$$L_{\nu,max} \simeq \frac{GM^2/R_{nuc}}{t_{col}} \simeq 10^{57} \text{ erg/sec} \quad (32)$$

↖ from (25)

Instead, because neutrinos escape only on time scale $t_{dif} \simeq 0.1\, \rho_{12}$ sec from (30)

the actual neutrino luminosity is

$$L_{\nu,act} \simeq \frac{GM^2/R_{nuc}}{t_{dif}} \simeq 10^{52} \text{ erg/s} \; ! \quad (33)$$

⇒ most of Gravitational binding energy is not immediately released as neutrinos, but first into heating and nuclear excitation, and core bounce kinetic energy. The late stages of core collapse proceed adiabatically. But can neutrinos ultimately drive the explosion? →

More on understanding Difficulty of Driving Type II SN with neutrinos

- when core density reaches several times $\rho_{nu} \simeq 3 \times 10^{14}\ g/cm^3$ core is stiff enough to halt the collapse. (As stated earlier, the core would be neutronized iron at this stage)
- outer collapsing core is in free fall and upon crashing into the stiff core can rebound somewhat
- this rebounding material acts as a piston that drives a shock into the layers above.
- the outward moving baryonic material carries of order $\simeq 10^{51}$ erg, some of which is thermalized at the shock via neutrino deposition and nuclear dissociation (fission)
- 10^{53} erg released directly as neutrinos
- this randomization of the bulk motion, or ultimately the drain of binding energy into neutrinos could be able to drive the SN explosion if the neutrinos could act as radiation pressure to drive off the outer layers of the star. But can they?
- To understand this, let us first look at how radiation pressure works for photons →

Eddington luminosity for photons

Consider the force exerted on ionized plasma via radiation. Assume that the dominant opacity is thompson scattering (photon scattering off of free electrons) with cross section $\sigma_T (\simeq 6.6 \times 10^{-25} cm)$.

Force must involve the cross section because the latter measures how effectively the matter and radiation couple.

A photon of momentum p deposits, on average, a momentum p to electron per scattering. Energy of photon is pc, so if all the photons are moving radially in a spherical system, the number of photons crossing a unit area per time is

$$\frac{F}{\mathcal{E}_\gamma} = \frac{L}{4\pi r^2 \mathcal{E}_\gamma} = \frac{L}{4\pi r^2 pc} \quad (34)$$

(flux → F; energy → $\mathcal{E}_\gamma$)

The number of scatterings per electron per time is determined by multiplying by the cross section

$$\Rightarrow N = \frac{L\sigma_T}{4\pi r^2 pc} \quad (35)$$

→

the Force per electron is the rate
at which momentum is deposited per unit time.
Since each photon transfers momentum p,
the force is then

$$f = Np = \frac{L\sigma_T}{4\pi r^2 c} \quad (36)$$

Now if this force is competing against gravity, we note that both (36) and grav force vary as $1/r^2$. Because electrons are coupled by coulomb collisions to protons for dense enough systems, the gravitational force on electrons is communicated through the protons. Thus force balance =>

mass of object within r

$$\frac{L\sigma_T}{4\pi r^2 c} = \frac{GMm_p}{r^2} \quad (37)$$

$$\Rightarrow L_{crit} \simeq \frac{4\pi c G M m_p}{\sigma_T} \simeq 1.3 \times 10^{38}\left(\frac{M}{M_\odot}\right)\frac{\text{erg}}{\text{s}} \quad (38)$$

for $L > L_{crit}$, radiation force can blow off outer layers of object.
Eq (38) is the standard Eddington Luminosity
note its dependence on the mass and σ_T
→

Now since neutrinos are also relativistic (here $\varepsilon_\nu \simeq 25$ MeV) in environments of neutronized stellar cores we can, by direct analogy to the photon case, derive an "Eddington neutrino luminosity"

$\Rightarrow$

$$L_{edd,\nu} \simeq \frac{4\pi G M c}{\kappa_\nu} \qquad (39)$$

$\uparrow$ where $\kappa_\nu = \frac{\sigma_A^{coh}}{A m_u}$

κ_ν is the dominant neutrino opacity (which has units of cross section per mass, and here $A m_u$ plays the role of m_p in (38))

$\uparrow$ atomic mass unit

Assuming that opacity in the mantle above the outward propagating shock is dominated by coherent scattering with neutrinos, from the equation below, and using $A \simeq 56$ ($Z=26$)

$$\kappa_\nu \simeq 2 \times 10^{-17} \, cm^2/g \qquad (40)$$

(* Z dependence ignored in rough estimate)

Using this in (39) $\Rightarrow$

$$L_{edd,\nu} \simeq 2 \times 10^{54} \, erg/sec \left(\frac{M}{M_\odot}\right) \qquad (41)$$

But this is larger than eqn (33)!

From the comparison, one can see that even if one could increase (33) by an order of magnitude and reduce (41) by an order of magnitude by changing assumptions, the values would still be close and given the complexity of the problem the debate rages on!

<u>What assumptions might one change?:</u>

- The calculation above assumes spherical symmetry. If the infall of material were not spherically symmetric, the neutrino luminosity could be larger in some directions if the t_{dif} in (33) could be made shorter and the mass required to be ejected along that direction reduced. (see also discussion on page 45)
- radiation pressure could help to remove some of the pre-existing envelope, reducing the burden on neutrinos.
- neutrino driven convection can reduce t_{dif} in (31)
- change the neutrino spectrum to take advantage of $\sigma_\nu \propto E_\nu^2$ (e.g. Ramirez-Ruiz & Socrates 2004, 2005) this increases $\langle \sigma_\nu n \rangle$ reducing (41)
- rotation: if one allows rotation one has natural asymmetries, additional energy sources from both differential rotation and total rotation. ⇒ mag. fields.

In General: core collapse requires understanding { neutrino transport, rotation, magnetic fields