

Fall 2008: WELCOME TO A231 EXAM 2

Do your best to answer each question. You may use calculators and any course materials except classmates.

Q1 (20 points): An asteroid starts its journey at a coordinate distance r_0 from a much more massive Schwarzschild black hole of mass M , such that $r_0 \gg 2M$. The impact parameter measured at r_0 is b . The asteroid takes a scattering orbit, approaching the hole at coordinate radius of nearest approach R , after which it is deflected away. Find the speed of the asteroid that a stationary observer at R would measure in terms of M , R and b .

Q2 (20 points): Using the definition of the general inner product, the chain rule, the geodesic equation, and the Christoffel symbol expressed in terms of the metric, show that the inner product of 4-velocities is a constant along a geodesic. That is, show $d(\mathbf{u} \cdot \mathbf{u})/d\tau = 0$ without specifying a priori the numerical value of the inner product.

Q3 (20 points): We derived the non-perturbative gravitational redshift in the Schwarzschild metric using the conserved quantity derived from *time*-translation symmetry. Derive the cosmological redshift for the Robertson-Walker metric, $ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$, using the analogous procedure that appeals to the *space*-translation symmetry. (This is not the method we used to get the cosmological redshift in class.)

Q4 (20 points): The formula for peak emission frequency (in Hz) for a black body of temperature T (in K) is given by $\omega = 6 \times 10^{10}T$. Calculate the peak emitted frequency for Schwarzschild black holes of $M = 10^9 M_\odot$ and $M = 10 M_\odot$ respectively, if both are accreting at rates needed to produce the 1/10 of the Eddington luminosity at the inner most stable orbit.

Q5: We derived the formula for $\frac{\omega_\infty}{\omega_{em}}$, the ratio of asymptotically observed to emitted frequency of photons from an edge-on view of emitting disk material in orbit around a Schwarzschild black hole. For material at $r = 6M$ we found the *minimum* ratio $\frac{\omega_\infty}{\omega_{em}} = \frac{\sqrt{2}}{3}$.

(a) (10 points): What is the *maximum* frequency ratio for photons emitted from material orbiting at $r = 6M$ and what azimuthal position of the orbiting material does this correspond to?

(b) (10 points): Discuss the different contributions to the frequency shift and the physical meaning of the maximum and minimum values.