PSET 1 SOLUTIONS AST232 F '04 (Eric Blackman)

new edition

1.1 (1.1) previous edition

$$L_\odot = 3.86 \times 10^{33}\ \text{erg/s} = 3.86 \times 10^{26}\ \text{W}$$

$$M_\odot = 2\times10^{33}\,\text{g} = 2\times10^{30}\,\text{kg}$$

$$\frac{L_\odot}{M_\odot} = 1.93 \times 10^{-4}\ \text{W/kg}$$

indeed about 10^{-4} that of Human output of 1 W/kg

1.2 (1.2)

(a) $L = 4\pi R_\odot^2 \sigma_{SB} T_{eff}^4$

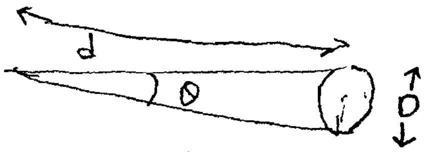
$$R_\odot = \left[\frac{L}{4\pi\sigma_{SB}T_{eff}^4}\right]^{1/2} \simeq \left[\frac{3.86\times10^{26}\ \text{W}}{(4\pi)(5.7\times10^{-8}\ \text{W/m}^2\text{K}^4)(5780\ \text{K})^4}\right]^{1/2}$$

$$\simeq \underline{7\times10^{8}\ \text{m}}$$

(b)
$$\frac{g_\odot}{g_\oplus} = \frac{g_{sun}}{g_{earth}} = \frac{\cancel{G} M_\odot/R_\odot^2}{\cancel{G} M_\oplus/R_\oplus^2} = \frac{M_\odot}{M_\oplus}\frac{R_\oplus^2}{R_\odot^2}$$

$$= \left(\frac{2\times10^{30}\ \text{kg}}{6\times10^{24}\ \text{kg}}\right)\left(\frac{6.4\times10^{6}\ \text{m}}{7\times10^{8}\ \text{m}}\right)^2 \simeq 28$$

$\boxed{1.3}$ (1.3) $T_{eff} = 3500\,K$



(I)

a) $\theta = 0.045'' = 0.045''\left(\frac{1^\circ}{3600''}\right)\left(\frac{2\pi\ \text{rad}}{360^\circ}\right) = 2.2\times10^{-7}\ \text{rad}$

$d = 140\,pc \simeq 4.13\times10^{20}\,cm$

$D = \theta d \simeq (4.13\times10^{20})(2.2\times10^{-7}) \simeq 9.3\times10^{13}\,cm$

$D = 2R \Rightarrow R \simeq 4.65\times10^{13}\,cm$

compare to $R_\odot = 7\times10^{10}\,cm$: $R/R_\odot \simeq 664$

b) $L = 4\pi R^2 \sigma_{SB} T_{eff}^4$

$\simeq (4\pi)(4.65\times10^{13}\,cm)^2(5.7\times10^{-5})(3500)^4$

$\simeq 2.3\times10^{38}\,erg/s \simeq 10^5 L_\odot$

$\boxed{1.4}$ (1.4)

a) $L_\odot = 3.9\times10^{26}\,W$

fusion converts 0.007 of H restmass into radiation

$\Rightarrow L_\odot = \frac{0.007\,Mc^2}{1\,sec} \Rightarrow M = \frac{L_\odot}{c^2\,0.007}$

$= \frac{3.9\times10^{26}\,W}{(9\times10^{16}\,m^2/s^2)\,0.007}$

$= 6.2\times10^{11}\,kg$

b) Suns mass in H is 70% of its total

$$\Rightarrow (0.7)(2\times10^{30}\ \mathrm{kg}) = 1.4\times10^{30}\ \mathrm{kg} = M_{\odot,H}$$

fusion efficiency

$$\tau_{\text{life}} \simeq \frac{(0.007)\, M_{\odot,H}\, c^2}{L_\odot} = \frac{(0.007)(1.4\times10^{30})(9\times10^{16})}{3.9\times10^{26}}$$

$$= 2.3\times10^{18}\ \mathrm{sec} = 7.5\times10^{10}\ \mathrm{yr}$$

$\boxed{1.5}$ (1.5)

$$L = 4\pi R^2 \sigma_{SB} T_{\text{eff}}^4$$

$$L_{ZAMS} = 0.69 L_\odot$$

present solar luminosity

$$\Rightarrow 0.69 L_\odot = 4\pi R_{ZAMS}^2 \sigma_{SB} T_{\text{eff}}^4$$

$$R_{ZAMS} = \left[\frac{0.69 L_\odot}{4\pi \sigma_{SB} T_{\text{eff}}^4}\right]^{1/2} = \left[\frac{(0.69)(3.9\times10^{26}\ \mathrm{W})}{(4\pi)(5.7\times10^{-8})(5640)^4}\right]^{1/2}$$

$$= 6.1\times10^{8}\ \mathrm{m}$$

presently, $R_\odot \simeq 7\times10^{8}\ \mathrm{m}$

$$\Rightarrow \frac{6.1}{7} \simeq 0.87$$

$\boxed{1\text{-}6}$ Assume distance to source >> binary separation
(1.6)

$m_1 = m_2 = m_*$ the two stars have equal apparent magnitudes
$F_1 = F_2 = F_*$ and equal fluxes

thus:

$$m_1 = m_{ref} - 2.5 \log_{10}\left(\frac{F_*}{F_{ref}}\right) = m_*$$

$$m_2 = m_{ref} - 2.5 \log_{10}\left(\frac{F_*}{F_{ref}}\right) = m_*$$

but taken together:

the apparent magnitude of the binary system is

$$m_{bin} = m_{ref} - 2.5 \log_{10}\left(\frac{2F_*}{F_{ref}}\right)$$

$$\Rightarrow m_{bin} - m_* = -2.5 \log_{10}(2F_*) + 2.5 \log_{10} F_*$$

$$= -2.5 \log_{10}(2) = -0.75$$

$\boxed{1\text{-}10}$ "when soure dimmed by $e^{-\tau_\lambda}$ show, using Eqn 1.10 that apparent mag increases by $A_\lambda = 1.086\tau_\lambda$"

(n/A)

$$m_1 - m_2 \equiv -2.5 \log F_1/F_2$$

let $F_2 = F_1 e^{-\tau_\lambda}$ =>

$$\Rightarrow m_1 - m_2 = -2.5 \log_{10} e^{+\tau_\lambda}$$

$$= -2.5\, \tau_\lambda \log_{10} e = -1.086\, \tau_\lambda$$

$A_\lambda = m_2 - m_1 = 1.086\,\tau_\lambda$: $\tau_\lambda > 0$ so

" $m_2 > m_1$ since larger apparent magnitude means dimmer source

$\boxed{1.11}$
(1-7)

a)

Earth Atmosphere density

$$\frac{6\times10^{23}}{(22.4_L)\left(10^3\frac{cm^3}{L}\right)} = \frac{2.7\times10^{19}}{cm^3} = n_{atm}$$

note: Earths atmosphere is mostly nitrogen wheras ISM is mostly hydrogen

for ISM $n_{ISM} \simeq \frac{1}{cm^3}$

to compress ISM to atmospheric density need to change inter particle spacing by factor of $\left(\frac{n_{atm}}{n_{ISM}}\right)^{1/3} \simeq (2.7\times10^{19})^{1/3}$ cm $= 30$ km

b) $r_g \sim 0.1\,\mu m = 10^{-5}$ cm

$\sigma_g \sim \pi r_g^2 \sim 3.14\times10^{-10}\,cm^2$

$n_g = \frac{n_{compressed}}{10^{12}} \sim (10^{-12})(2.7\times10^{19}) \simeq 2.7\times10^{7}\,/cm^3$

$L = 1cm$

$\Rightarrow$ optical depth : $\tau = n_g\sigma_g L = (2.7\times10^7)(3.14\times10^{-10})(1)$
$= 0.0085$

$\Rightarrow \frac{F}{F_0} = e^{-\tau} = e^{-0.0085} \simeq 0.99$

$\Rightarrow$ 1% absorbed

• note that the length for which $\tau = 1$ is: $\tau = 1 = n_g\sigma_g\tilde{L} \Rightarrow \tilde{L} = \frac{1}{n_g\sigma_g} \simeq \frac{1}{0.0085} = 118$ cm

not < 1m as stated in text