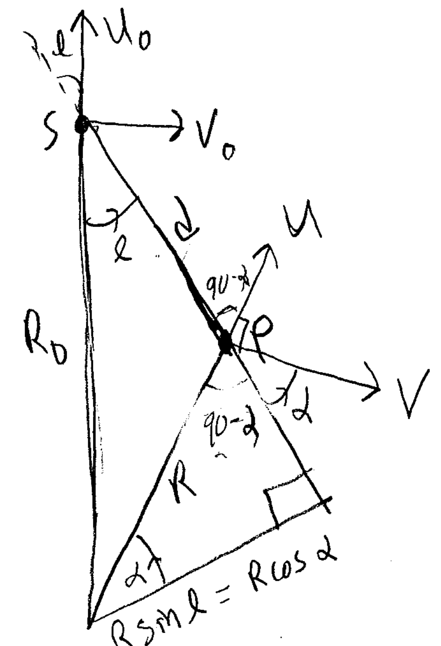
Fall '09 — PSET 3 SOLN's — Eric Blackman — ①

AST 232W

2.16)

a)



· Radial velocity of P is away from S parallel to d:

$$V_r = V\cos\alpha - V_0\sin\ell - u\underbrace{\cos(90°-\alpha)}_{\sin\alpha} + u_0\cos\ell$$

$$= V\cos\alpha - V_0\sin\ell - u\sin\alpha + u_0\cos\ell \qquad (1)$$

use: $R_0\cos\ell = d + R\sin\alpha$

$$\Rightarrow \sin\alpha = \frac{R_0\cos\ell - d}{R} \qquad (2)$$

use $R_0\sin\ell = R\cos\alpha$ from figure

so that $\cos\alpha = \frac{R_0}{R}\sin\ell \qquad (3)$

plug (2) & (3) into (1):

$$\Rightarrow V_r = V\frac{R_0}{R}\sin\ell - V_0\sin\ell - u\left(\frac{R_0\cos\ell - d}{R}\right) + u_0\cos\ell$$

$$= \boxed{R_0\sin\ell\left(\frac{V}{R} - \frac{V_0}{R_0}\right) - R_0\cos\ell\left(\frac{u}{R} - \frac{u_0}{R_0}\right) + \frac{du}{R}} \qquad (4)$$

b) • velocities measured in direction of $l = 180°$ will be "negative": toward us

c) to find extrema take

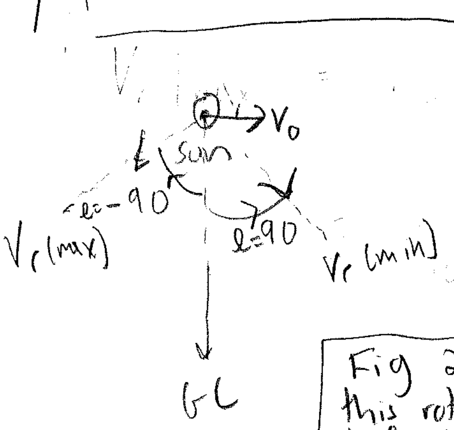
$$\frac{dV_r}{dl} = 0 \Rightarrow \tan l = -\frac{\left(\frac{V}{R} - \frac{V_0}{R_0}\right)}{\frac{u}{R} - \frac{u_0}{R_0}}$$

for $u_0 > 0$ and $u = 0$

$$\Rightarrow \tan l_{ex} = \frac{+\left(\frac{V}{R} - \frac{V_0}{R_0}\right)}{\frac{u_0}{R_0}} = \frac{\sin l_{ex}}{\cos l_{ex}} \qquad (5)$$

• this can be compared with extrema of equation 2.11 which are at $\cos l \big|_{u=u_0=0} = 0$ or $l = 90, 270$. At $l = 270$ $V_r\big|_{u=u_0=0}$ is maximum (positive) and at $l = 90$ $V_r\big|_{u=u_0=0}$ is minimum (negative).

d) case with $u = u_0 = 0$: | case with $u = 0, u_0 > 0$:



(minimum and max are rotated counter clockwise when $u_0 > 0$)

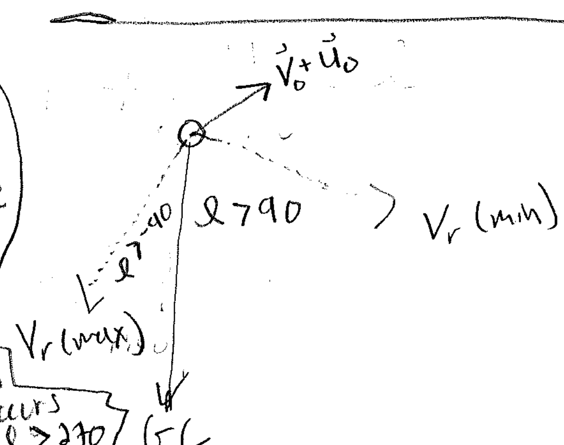


Fig 22 is consistent with this rotation: minimum V_r occurs at $l > 90$; maximum occurs at $l > 270$

2.22)

a) $$\lambda_{max} = \frac{2.9}{T(K)} \text{ mm} \quad \text{(Wien's law)}$$

$$\lambda_{max} = 150\,\mu m = 0.15\,mm$$

$$\Rightarrow T = \frac{2.9}{0.15} = \boxed{19.3\ K}$$

b) $$L = 10^6 L_\odot = 3.9\times10^{39}\ erg/s$$

$$d = 1pc \simeq 3\times10^{18}\ cm$$

- Flux received $= \frac{L}{4\pi(3\times10^{18})^2} \simeq 35\ erg/cm^2\text{-}s = F_r$

- energy/time received $= \pi r_g^2 F_r$

- in equilibrium energy/time received equals luminosity emitted by grain:

$$\Rightarrow \underbrace{\cancel{\pi}\cancel{r_g^2} F_r}_{L_r} = \underbrace{4\cancel{\pi}\cancel{r_g^2}}_{L_e}\sigma_{SB}T^4 \quad \text{Stefan-Boltzmann}$$

$$\Rightarrow T = \left(\frac{F_r}{4\sigma_{SB}}\right)^{1/4} = \left(\frac{35}{(4)\,5.7\times10^{-5}}\right)^{1/4}$$

$$\simeq \boxed{19.7\ K}$$

c) From Fig 2.24,

$$\lambda_{max} \approx 60\,\mu m \approx 0.06\,mm$$

$$\lambda_{max} = \frac{2.9}{T}\,mm \Rightarrow T = \frac{2.9}{0.06} = 48.3\,K$$

d) energy per unit time absorbed = energy per unit time radiated, as in part b

$$(10^6 L_\odot)\frac{\pi \cancel{r_g^2}}{4\pi d^2} = 4\pi \cancel{r_g^2}\,\sigma_{SB} T^4$$

$$d = \left(\frac{10^6 L_\odot}{16\pi\sigma_{SB}T^4}\right)^{1/2} = \left(\frac{3.9\times10^{39}\ erg/s}{(16)(\pi)(5.7\times10^{-5})(150\,K)^4}\right)^{1/2}$$

$$= 5\times10^{16}\,cm \approx \boxed{0.02\,pc}$$

(note that $T = 150\,K$ corresponds to $\lambda_{max} = \frac{2.9}{150\,K} \approx 0.02\,mm \approx 20\,\mu m$ not $30\,\mu m$ as stated)

2.23)

$T = 50\,K$

a) $\frac{m_p V_{th}^2}{\cancel{2}} = \frac{3 k_B T}{\cancel{2}}$

$$V_{th} = \left(\frac{3 k_B T}{m_p}\right)^{1/2} = \left(\frac{(3)(1.4\times10^{-16})\,50}{1.6\times10^{-24}\,g}\right)^{1/2}$$

$$= 1.1\times10^5\ cm/s = 1.1\ km/s$$

b) $a = 0.1\,\mu m = 10^{-5}\,cm$

$n_g = 10^{-12}\,n_H$

Consider H atom passing through the medium: the collision frequency with dust grains is $n_g \sigma_g V_{th}$. This follows because $n_g \sigma_g$ is the number of dust particles per unit length and V_{th} is the speed the H atom travels (length per time)

number of grains per volume

dust cross section area πa^2

H atom speed

So $n_g \sigma_g V_{th}$ is number of grains per unit time that each H atom encounters

- The time between encounters is then (assuming interaction time is short)

$$\frac{1}{n_g \sigma_g V_{th}} = \tau = \frac{1}{(10^{-12} n_H)(10^{-5}cm)^2 \pi (1.1\times10^5 cm/s)}$$

$$= 2.9\times10^{16}\ sec \left(\frac{n_H}{100\,cm^{-3}}\right)^{-1}$$

note 100 Not 1 as in text book

$$= 9.7\times10^6\ yr \left(\frac{n_H}{100\,cm^{-3}}\right)^{-1}$$

→

c) passing through spiral arm takes

$\sim (0.1) 240\ \text{Myr} = 24\ \text{Myr}$

to produce H_2 on grains we need

τ from part b to be $\tau < 24\ \text{Myr}$

This is satisfied for $n_H = 100/\text{cm}^3$ as seen in answer to b).

2.24)

$$\text{gas mass} = \frac{4\pi m_p r_*^3 n_H}{3}$$

$$S_{*,0} = \frac{4\pi r_*^3}{3} n_H^2 \alpha(T_e) = \frac{M_g}{m_p} n_H \alpha(T_e) \quad (1)$$

m_p proton mass

a) for $S_{*,0} = 10^{49}/\text{s}$, $n_H = 10^3/\text{cm}^3$, $T_e = 10^4\ \text{K}$:

$\alpha(T_e) \simeq 2\times10^{-13}\ \text{cm}^3/\text{s}$ at $T_e = 10^4\ \text{K}$ from (1)

$$\Rightarrow r_{*,0} = \left(\frac{3 S_*}{4\pi n_H^2 \alpha}\right)^{1/3} = \left(\frac{3\times10^{49}}{4\pi(10^3)^2\, 2\times10^{-13}}\right)^{1/3} \quad (2)$$

$$= 2.3\times10^{18}\ \text{cm} \simeq \underline{\underline{0.7\ \text{pc}}}$$

b) for n_H 10 times larger

r_* decreases by $\frac{1}{10^{2/3}}$ factor $\Rightarrow$

$$\boxed{0.16 \text{ pc}}$$

Eqn (1) on previous page shows that $M_g \propto n_H$ so $\underline{1/10}$ as much mass is ionized

c) for $S_{*,B} = 3 \times 10^{47}$, $n_H = 10^3$

we can use the result of part a) to scale:

$$r_{*,B1} = r_{*,0} \underbrace{\left(\frac{3 \times 10^{47}}{10^{49}}\right)^{1/3}}_{\frac{S_{*,B}}{S_{*,0}}}$$

$$= 0.3\, r_{*,0}$$

$$\boxed{\simeq 0.22 \text{ pc}}$$