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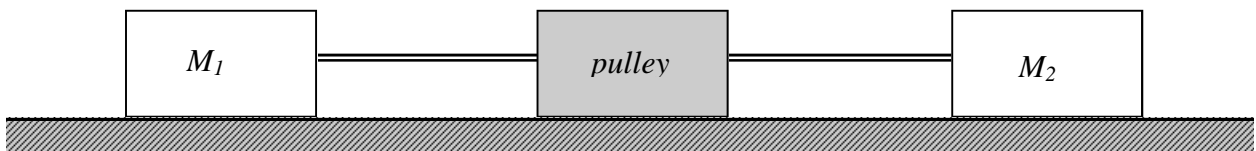
Last Revised on August 29, 2007

# EXPERIMENT 2

## Acceleration of Gravity

### 0. Pre-Laboratory Work [2pts]

1. You have just completed the first part of this lab and have five time values for a particular height: 1.8, 1.7, 1.9, 0.8, and 1.9 seconds. The value of 0.8 seconds is not consistent with the other measurements. Explain, quantitatively, why it is not. Provide a plausible explanation for what could have gone wrong in the 0.8 second trial (use details of the experimental apparatus). (1pt)
  
2. Because pulleys change the direction of forces it is often difficult to draw free body diagrams for systems with pulleys. This is especially true when we try to include friction. For that reason it is helpful to imagine Atwood's Machine as a tug-of-war on a flat plane. At the ends of a rope are two teams (representing the two masses) pulling in opposite directions. Think of the pulley as a block attached to the middle of the string, moving along the plane. The friction between the plane and the block represents the friction in the pulley. Assume that  $M_1$  and  $M_2$  do not experience friction from moving along the plane. Add on the forces of the system in the diagram below. (1pt)



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# Experiment 2

## Acceleration of Gravity

### 1. Purpose

The purpose of this experiment is to demonstrate how imperfections in the experimental apparatus can play a large role in the final results. Friction and rotational inertia intrinsic in the pulley have a significant effect. These extra forces mean that the acceleration of the masses is not actually the acceleration found using *Equation 2.1*. You will observe and quantify these effects. You will measure the acceleration due to friction in the pulley bearings and the effects of the rotation of the pulley disk.

In this version of the experiment, two digital photogate timers will measure the acceleration of the masses. Hand timers will also be used to understand the role that “better,” high speed equipment plays. The acceleration here at the River Campus is  $9.8039 \frac{m}{s^2}$ . All measurements can be compared to this value.

### 2. Introduction

Atwood's machine was originally designed by George Atwood in 1784 as an experiment demonstrating the effects of uniform acceleration. Atwood's machine reduces the acceleration of the masses to a fraction of the value of gravitational acceleration, and the lower acceleration is measured to greater precision than the unchanged acceleration of gravity with the same timing device. The smaller value of the acceleration is:

$$a = \frac{M_1 - M_2}{M_1 + M_2} g . \quad \text{Equation 2.1}$$

### 3. Laboratory Work [18 pts]

#### 3.1 The Equation of Motion for the Atwood's Machine

To find the equation of motion for Atwood's machine we calculate the sum of the forces acting on the system. There are three separate forces in our system: the force of gravity on each mass and the force due to friction in the pulley. The force due to friction on the pulley is the sum of the tension in the strings on either side of the pulley ( $T_1$  and  $T_2$ ) and the weight of the pulley ( $M_p g$ ) multiplied by  $\mu$ , the coefficient of friction.

$$\sum F = M_1 g - M_2 g - \mu(T_1 + T_2 + M_p g) = M_T a \quad \text{Equation 3.1}$$

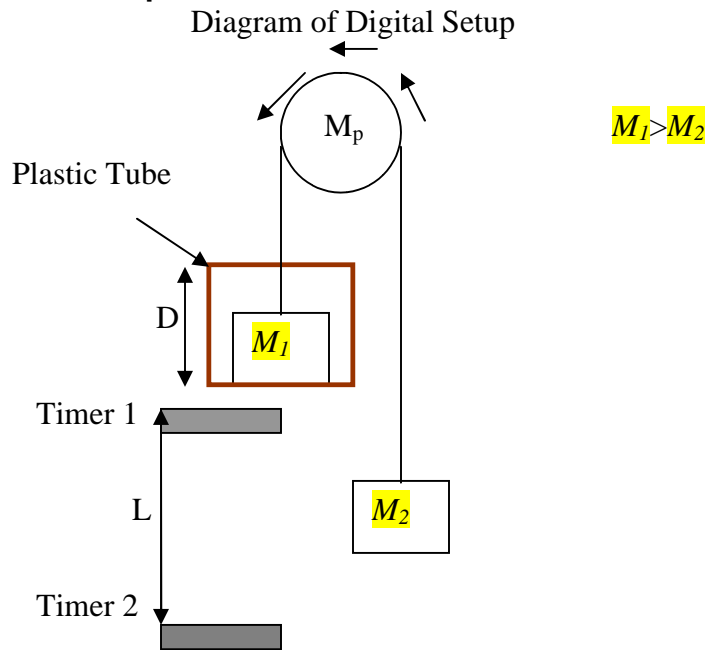
Note that we have defined the direction that the larger mass ( $M_1$ ) moves the positive direction.  $M_T$  is the total mass of the system and  $a$  is the acceleration. The total mass of Atwood's machine is made up of three separate masses:  $M_1$ ,  $M_2$  and  $M_p$ . Summing everything up we get the equation:

$$M_1 g - M_2 g - \mu(T_1 + T_2 + M_p g) = (M_1 + M_2 + \frac{M_p}{2})a. \quad \text{Equation 3.2}$$

$M_p$  is multiplied by a factor of  $\frac{1}{2}$  because the pulley is being angularly accelerated rather than linearly accelerated like  $M_1$  and  $M_2$ .

#### 3.2 Measuring the Acceleration

##### Digital Timer Setup



There are four variables that need to be measured to find the acceleration of gravity from this setup:  $D$ , the length of the plastic tube,  $L$ , the distance between the two photogate timers and  $t_1$  and  $t_2$ , the times recorded by the timers. The time recorded by a photogate timer is the amount of time there is an object inside the photogate, so in this situation  $t_1$  and  $t_2$  are the times it takes the plastic tube to fall through the first and second photogates, respectively. Speed is equal to distance divided by time, so at the first timer, the speed is

$$v_1 = \frac{D}{t_1}. \quad \text{Equation 3.3a}$$

For the second timer, the speed is

$$v_2 = \frac{D}{t_2}. \quad \text{Equation 3.3b}$$

The change between  $v_1$  and  $v_2$  is the acceleration. Using the kinematic equations, the acceleration can now be determined:

$$v_2 = v_1 + at \quad \text{Equation 3.4a}$$

$$L = v_1 t + \frac{1}{2} at^2 \quad \text{Equation 3.4b}$$

From *Equation 3.4a*:

$$t = \frac{v_1 - v_2}{a} \quad \text{Equation 3.5}$$

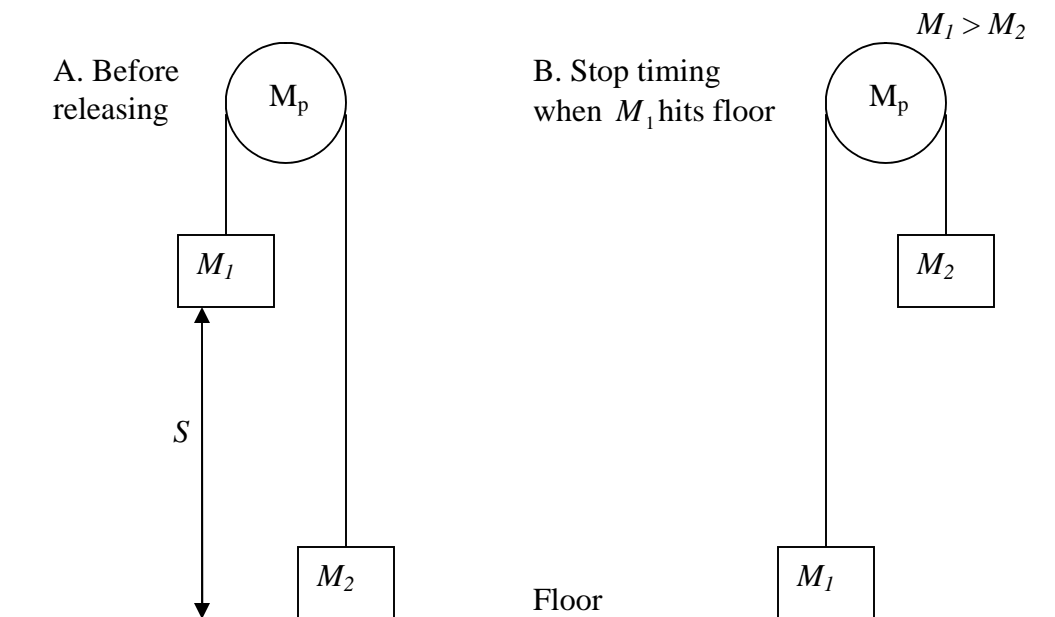
Combining *Equation 3.4b* and *Equation 3.5*:

$$\frac{v_1(v_2 - v_1)}{a} + \frac{a(v_2 - v_1)^2}{2a^2} = L \quad \text{Equation 3.6}$$

$$a = \frac{D^2}{2L} \left( \frac{1}{t_2^2} - \frac{1}{t_1^2} \right) \quad \text{Equation 3.7}$$

### Analog (Hand) Timer Setup

Diagram of Analog Setup



**Figure 3.2**

In this section  $M_1$  starts at rest, making the kinematics equations much simpler. Now only two variables must be measured to find the acceleration of gravity. The distance from the bottom of  $M_1$  to the floor is  $S$ . The time,  $t$ , is measured with the hand timer.

The kinematic equations for this section are similar to the previous section:

$$v_{final} = at \quad \text{Equation 3.8a}$$

$$y_{final} = S = \frac{1}{2}at^2 \quad \text{Equation 3.8b}$$

Solving for  $a$  we find:

$$a = \frac{2S}{t^2} \quad \text{Equation 3.9}$$

## 4. Measuring Acceleration Using Hand Timers and Photogates

In this section of the lab, photogate and hand timers are used to time the falling mass. In both methods, the acceleration of gravity will be measured using Atwood's Machine. The accuracy and the precision of these two methods will be compared. Equations developed in the introduction for finding the acceleration will be used in this section, as will the statistical techniques from the previous lab.

### 4.1 Procedure for Photogate Timers

1. Measure the mass of the plastic tube,  $M_D$ , and its length,  $D$ .
2. Set the photogates successively along the path of the mass. (Does it matter which of the masses passes through the photogates?)
3. Measure the distance,  $L$ , from the top of one of the photogates to the top of the other photogate. Write this distance in the table below.
4. Place the plastic tube around the mass that will be falling through the photogates.
5. Make  $M_1 = 260$  g. *Remember to include the mass of the plastic tube in this measurement.* Make  $M_2 = 240$  g.
6. Place the lighter mass on the floor each time, to ensure that initial conditions are the same, and after *resetting the photogate timers*, let the masses drop. Do not push the masses. (You want to measure the acceleration of gravity, and a push by anything other than gravity will skew the data.) Hold the pulley and not the masses to keep them steady for the fall.
7. Record  $t_1$  and  $t_2$  in the table below.  $t_1$  should be greater than  $t_2$ . (Why?)
8. Repeat steps 6 and 7 *ten* times for the mass pair to get a good estimate of the average value of  $g$  and the standard deviation (uncertainty) involved with timing.

$D =$  \_\_\_\_\_

$L =$  \_\_\_\_\_

$M_D =$  \_\_\_\_\_

$M_1 = M_D +$  \_\_\_\_\_  $= 260$  g

$M_2 = 240$  g

| Trial # | $t_1$ (sec) | $t_2$ (sec) | Trial # | $t_1$ (sec) | $t_2$ (sec) |
|---------|-------------|-------------|---------|-------------|-------------|
| 1       |             |             | 2       |             |             |
| 3       |             |             | 4       |             |             |
| 5       |             |             | 6       |             |             |
| 7       |             |             | 8       |             |             |
| 9       |             |             | 10      |             |             |

## 4.2 Procedure for Using Hand Timers

1. When measuring the displacement of the mass hanger  $S$ , let the heavier mass  $M_1$  touch the floor and use a 2 meter ruler to measure the distance from the floor to the bottom of the lighter mass holder. Place a level against the ruler to ensure that the ruler is perpendicular to the floor. Do not hold the masses or the string when measuring; otherwise, your measured height does not truly reflect the displacement for the masses.
2. Set  $M_1$  to 260 g (you may leave the plastic tube on) and  $M_2$  to 240 g, the same as they were for the digital timers. Practice dropping the masses a couple times to get a feel for how quickly  $M_1$  falls. Put your hand on the pulley, not the mass, to hold them steady before you release them.
3. Use a hand timer to make ten measurements of the time it takes for the heavier mass  $M_1$  to travel the distance  $S$  downward, and record them in the table below. It is best if the same person who starts and stops the timer also releases the mass. The other person should measure the distance.

$$M_1 = 260 \text{ g}$$

$$M_2 = 240 \text{ g}$$

$$S = \underline{\hspace{2cm}}$$

| Trial # | Time (sec) | Trial # | Time (sec) |
|---------|------------|---------|------------|
| 1       |            | 2       |            |
| 3       |            | 4       |            |
| 5       |            | 6       |            |
| 7       |            | 8       |            |
| 9       |            | 10      |            |

## Questions [9 pts]

*Answer Questions 1-3 using the photogate timer data:*

1. Calculate the average times:  $\bar{t}_1$  and  $\bar{t}_2$ . Also compute the standard deviation for  $\bar{t}_1$  and  $\bar{t}_2$ . Show at least one example for the calculation of both the average and standard deviation. (1 pt)

$$\bar{t}_1 = \underline{\hspace{2cm}}$$

$$\bar{t}_2 = \underline{\hspace{2cm}}$$

$$\Delta t_1 = \underline{\hspace{2cm}}$$

$$\Delta t_2 = \underline{\hspace{2cm}}$$

2. Find the average acceleration using *Equation 3.7*. Use the average times that you found in Question 1. (1 pt)

$$\bar{a} = \underline{\hspace{2cm}}$$

3. Using reasonable estimates for the error in  $L$  and  $D$ , find the uncertainty in the acceleration measured by photogate timers. Circle or underline the term that is the largest source of uncertainty. (1 pt)

$$\frac{\Delta a}{\bar{a}} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{2\Delta t_1}{t_1}\right)^2 + \left(\frac{2\Delta t_2}{t_2}\right)^2 + \left(\frac{2\Delta D}{D}\right)^2}$$

$$\Delta L = \underline{\hspace{2cm}}$$

$$\Delta D = \underline{\hspace{2cm}}$$

$$\Delta a = \underline{\hspace{2cm}}$$

**Answer Questions 4-6 using the hand timer data:**

4. Calculate the mean and standard deviation of  $t$ . (1 pt)

$$\bar{t} = \underline{\hspace{2cm}}$$

$$\Delta t = \underline{\hspace{2cm}}$$

5. Find the average acceleration using *Equation 3.9*. Enter the average time that you found in Question 4. (1 pt)

$$\bar{a} = \underline{\hspace{2cm}}$$

6. Using  $\Delta t$  and a reasonable estimate for the uncertainty in  $S$ , find the uncertainty for the acceleration measured using hand timers. The equation for summing these uncertainties is below. Show your work and circle or underline the term that is the largest source of uncertainty. (1 pt)

$$\frac{\Delta a}{\bar{a}} = \sqrt{\left(\frac{\Delta S}{S}\right)^2 + \left(\frac{2\Delta t}{t}\right)^2} \quad \Delta S = \underline{\hspace{2cm}}$$

$$\Delta a = \underline{\hspace{2cm}}$$

7. Use *Equation 2.1* to calculate  $g$  measured using photogates and hand timers. Also calculate the uncertainty in each method using the equation:  $\Delta g = \frac{M_1 + M_2}{M_1 - M_2} \Delta a$ .

Compare your answer from each method to the accepted answer of  $9.8039 \frac{m}{s^2}$  given in the introduction. (1 pt)

$$g_{photogate} = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}$$

$$g_{hand\ timer} = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}}$$

8. Compare and contrast the methods of timing by hand and timing using the photogates. Which method resulted in a larger uncertainty? What is the difference between the mean values for each method? Are they within a few standard deviations of each other? Using this information, decide which method is more precise. Is one method more accurate than the other? Support your answers with your data. (2 pts)

## 5. Correcting for the Mass of the Pulley

When you try to move an object in a straight line it is clear that the higher the mass of the object the harder you have to push in order to get the same acceleration. A similar rule applies to spinning objects. A car wheel is harder to spin than a bike wheel because the car wheel is much heavier. Spinning objects are more complicated because their motion depends not only on their mass, but also how the mass is distributed around the axis of rotation. You will deal with this complication formally later in the course; for now we will confine ourselves to relatively simple objects.

In the previous section you calculated the acceleration of the masses assuming that the pulley had zero mass. However, the pulley does have mass, and is accelerated as the masses fall. Does the acceleration of the pulley affect your measurement? This section will quantitatively determine the effect of the mass of the pulley on your measurement of  $g$ .

### 5.1 Procedure for Measuring Rotational Inertia

1. Replace the current pulley system with the large disc pulley.
2. Measure the distance between the floor and the bottom of the lighter mass as you did in the previous sections and record the result in below.
3. For each combination of  $M_1$  and  $M_2$  listed in the table below and measure the drop time once using the hand timers.
4. Record the mass of one metal ( $M_R$ ) ring below. Screw one metal ring to the pulley and repeat Step 3.
5. Add another metal ring to the pulley (there should be two rings on the pulley now) and repeat Step 3.

| $M_1$ | $M_2$ | Fall Time    |            |             |
|-------|-------|--------------|------------|-------------|
|       |       | 1) $M_R = 0$ | 2) $M_R =$ | 3) $2M_R =$ |
| 100 g | 105 g |              |            |             |
| 100 g | 110 g |              |            |             |
| 100 g | 115 g |              |            |             |

$S =$  \_\_\_\_\_

$M_R =$  \_\_\_\_\_

### Questions [6 pts]

1. Calculate  $\frac{M_2 - M_1}{M_1 + M_2}$  for each combination and place your answers in the table below.

Also calculate the acceleration for each trial using *Equation 3.9*. Show an example calculation below. (1 pt)

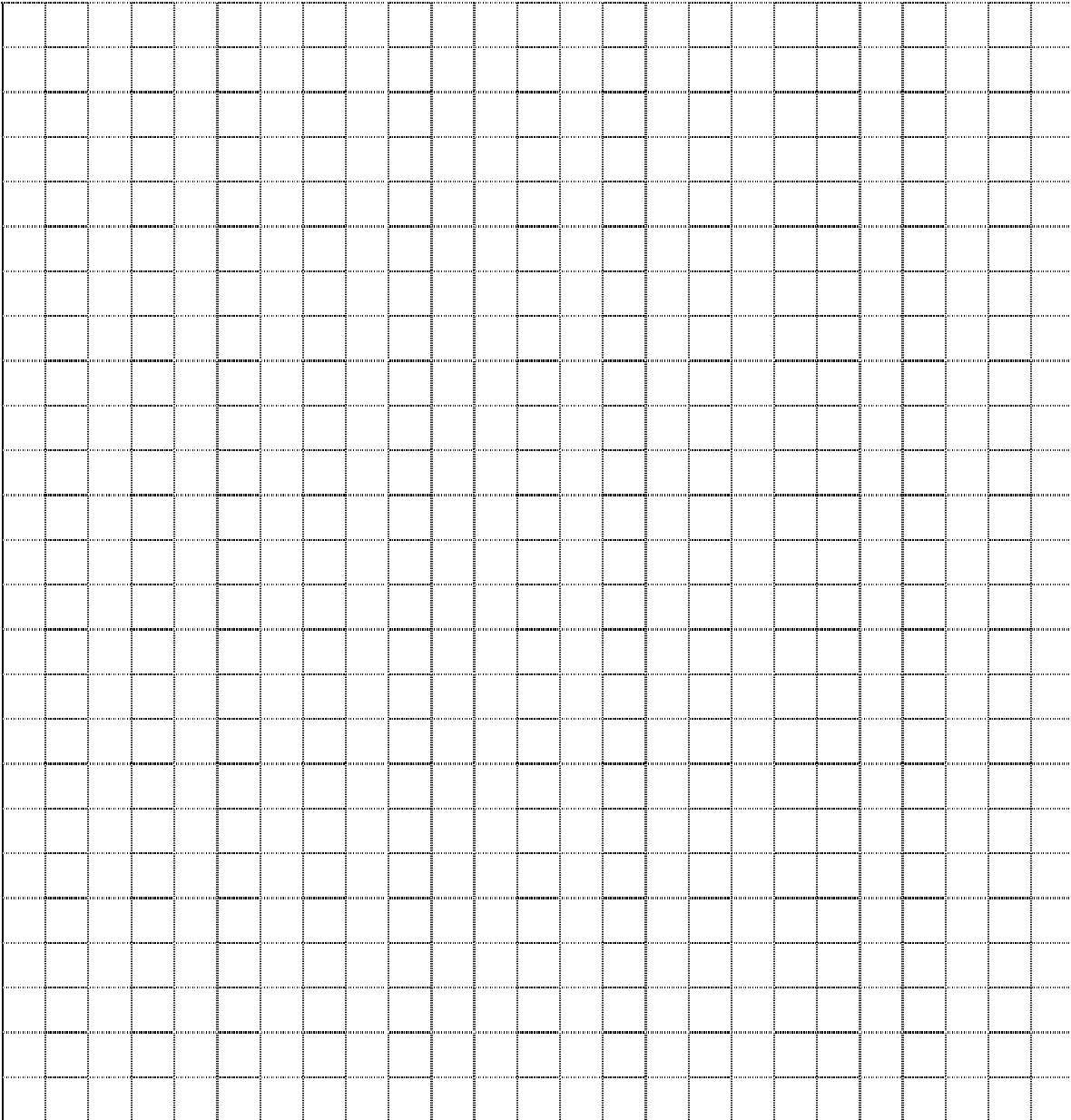
| $\frac{M_2 - M_1}{M_1 + M_2}$ | Acceleration<br>(0 rings) | Acceleration<br>(1 ring) | Acceleration<br>(2 rings) |
|-------------------------------|---------------------------|--------------------------|---------------------------|
|                               |                           |                          |                           |
|                               |                           |                          |                           |
|                               |                           |                          |                           |

Assuming that the mass of the pulley is negligible the acceleration of  $M_1$  is

$$a = \frac{M_1 - M_2}{M_1 + M_2} g \quad \text{Equation 5.1}$$

Thus a plot of  $a$  vs.  $\frac{M_1 - M_2}{M_1 + M_2}$  should result in a straight line with a slope equal to  $g$ .

2. Plot  $a$  vs. the mass ratio  $\frac{M_1 - M_2}{M_1 + M_2}$  (y vs. x) for each set of three data points. Make sure to include a title for your plot and labels for your axes. Also draw a legend to label your data. It should be clear which data points correspond to which pulley mass. (2 pts)



3. How does your measurement of  $g$  change as the mass of the pulley increases? *Hint: Try adding an appropriate best fit line to your data. Also, remember that  $g$  is proportional to the slope.* (1 pt)

4. Given the results from this section should you have included the mass of the pulley in the first section? Consider the differences between the original pulley system and this one and be as quantitative as possible in your answer. (2 pts)

## 6. Summary Questions [6 pts]

1. What is another significant source of error in Atwood's machine not investigated in the lab? Explain your reasoning. (2 pts)

2. What improvements could you make on Atwood's machine in order to make the results more accurate? What improvements would make the results more precise? Carefully explain your answer and use diagrams if necessary. (2 pts)

3. The presence of a pulley that has mass and friction introduces significant error into the measurements made using Atwood's Machine. Given that Atwood's machine contains so much systematic error why was it used instead of directly measuring the acceleration of an object in freefall? (2 pts)