...cretion with Angular momentum: Lessons from stellar binary systems

- binaries, (in particular X-ray binaries) are where we have learned <u>a lot</u> about accretion: why binaries?

- orbiting system, tidal forces
  $\Rightarrow$ shearing of material into a disk; to accrete, ∡ momentum must be shed (or equivalently, transported outward)

two reasons for binary mass transfer via accretion:

(1) one of the stars may increase in size during evolution: companion can rip off outer layers

(2) ejection of mass by stellar wind

Important concept is <u>Roche lobe overflow</u>

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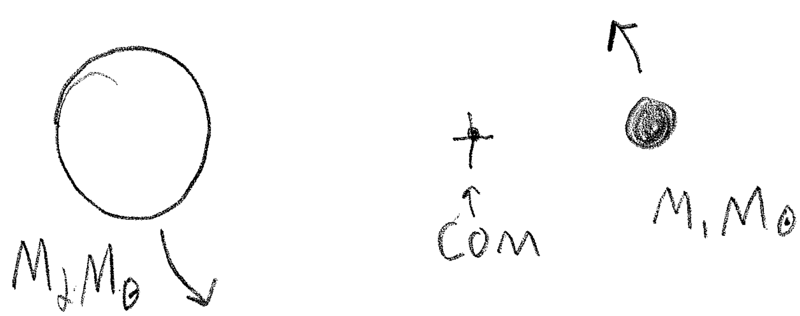
- 19th century Edouard Roche studied destruction of planetary satellites (moons, etc.)

- Basic idea was to consider orbit of test particle in grav potential of two orbiting, massive bodies

- Assume two "centrally condensed" stars orbit each other in keplerian, circular orbits, and consider test particle gas motion in the potential (also called "restricted 3-body problem", because gas is assumed not to influence the binary orbit)

- 

- Gas flow between stars governed by Euler equation. In rotating frame:

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} = \underbrace{-\vec{\nabla}\phi_R}_{\text{Grav + Cent. force / mass}} - \underbrace{2\vec{\omega} \times \vec{v}}_{\text{coriolis force / mass}} - \frac{1}{\rho}\nabla P$$

$\vec{\omega}$ from Kepler's law $\vec{\omega} = \omega\hat{z}$:

$$\vec{\omega} = \left[\frac{G(M_1+M_2)}{a^3}\right]^{1/2}\hat{z}$$

normal to the orbit plane, a = binary separation

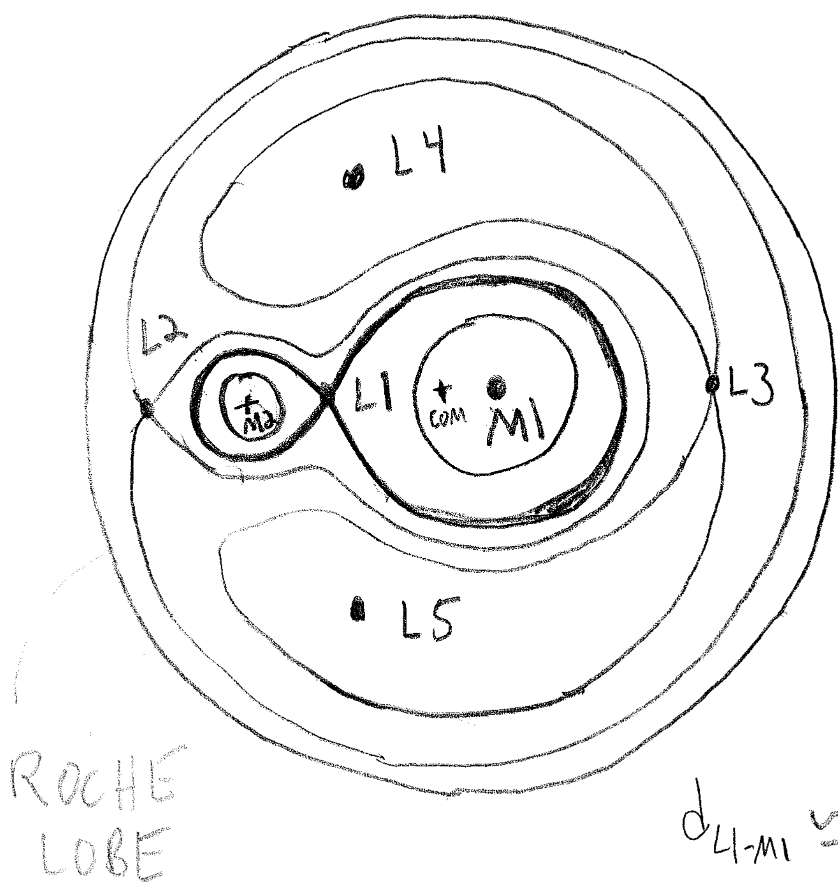
$$\phi_R = -\frac{GM_1}{|\vec{r}-\vec{r}_1|} - \frac{GM_2}{|\vec{r}-\vec{r}_2|} - \frac{1}{2}(\vec{\omega}\times\vec{r})^2$$

↑ roche potential.

Set ϕ_R = constant and plot:

L4, L5 maxima, (but coriolis force stabilizes)

L1, saddle → material can overflow if e.g. M2 fills lobe, it can accrete onto M1



note: distance $\underbrace{\text{L1 ——— M1}}$

$$d_{L1\text{-}M1} \simeq \frac{1}{2}a - 0.23\,a\log_{10}\frac{M_2}{M_1}$$

agine evolution:

1) assume both stars are smaller than Roche lobe and are in circular orbit, and tidally locked

$\Rightarrow$ surface of each star corresponds to circular equipotential surface. (follows from momentum equation with $\vec{v}=0$ and $\nabla p = 0$ on surface of star)

$\hookrightarrow$ Binary is fully detached

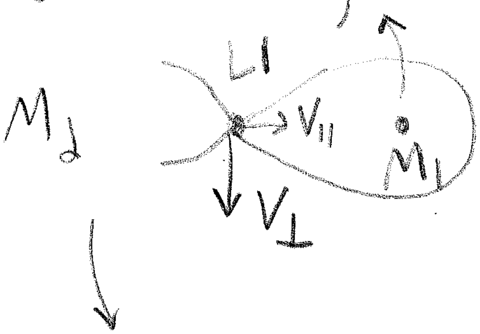
2) If one star then swells & fills Roche lobe (usually called secondary star) then primary can accrete this is a semi-detached binary

3) can you guess what a contact binary is?

(both stars fill Roche lobes)

Formation of Disk in Binary

Note that material is pushed through Roche lobe at $\simeq c_s$, but because of orbit, $V_\parallel << V_\perp$ for typical systems.



→ remember, previous equations are in rotating frame about c.o.m; as soon as material edges over L1, M_1 sees it in orbit.

$$V_\parallel \simeq c_s \; ; \; V_\perp \simeq V_K$$

typically $V_\parallel << V_\perp$, [$c_s \sim 10$ km/s $(T/10^4 K)$

$$V_\perp \simeq 10^2 M_1^{1/3}\left(1+\frac{M_2}{M_1}\right)^{1/3} P_{day}^{-1/3} \text{ km/s} \;]$$

from Kepler's law

⇒ gas has ∡ momentum which it needs to shed to accrete

⇒

Gas will first orbit in circle at

at $V_\phi(R_{circ}) = \left(\frac{GM_1}{R_{circ}}\right)^{1/2}$

L1–M1 distance

with $V_\phi(R_{circ})\, R_{circ} = \left(d_{L1\text{-}M1}^2\, \omega\right)$ orbit velocity

$\underbrace{}$ ∡ momentum conservation

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using kepler: $\left(\begin{array}{l} 4\pi^2 a^3 = G(M_1+M_2)\,P^2 \\ \text{and } P = \frac{2\pi}{\omega} \end{array}\right.$ and $V_\phi(R_{circ}) = \left(\frac{GM_1}{R_{circ}}\right)^{1/2}$

and formula for $d_{L1\text{-}M1}$ on page 134 ⇒

→ $R_{circ}/a = \left(\frac{4\pi^2}{GM_1 P^2}\right) a^3 \left(d_{L1\text{-}M1}/a\right)^4 =$

⇒ $= R_{circ}/a = \left(1+\frac{M_2}{M_1}\right)\left(\frac{1}{2} - 0.23\log_{10}\frac{M_2}{M_1}\right)^4$

(it is possible to have $R_{circ} < R_{*,1}$ ← size of M_1, but never true for NS, BH, or WD systems)

So we have gas orbiting at $R = R_{circ}$, but thats not the end of the story!

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Internal dissipation will lead to radiation → loss of radiation => loss of kinetic energy => material sinks deeper into grav. potential => accretion => loss of ∡ momentum.

→ Now t_{cool} (cooling time) is usually $\ll t_{acc}$ and $t_{dyn} \ll t_{acc}$ so that material spirals in slowly

→ but if material loses ∡ momentum what carries it? : some material actually goes outward, so "initial ring" turns into disk.

→ usually for compact objects disk is not self gravitating ($\rho \ll M/R^3$)

$\Rightarrow \quad \Omega \simeq \Omega_K = \left(\frac{GM_1}{R^3}\right)^{1/2}$ Keplerian orbits

⟶

→ kinetic energy of gas element Δm in keplerian orbit is

$\frac{1}{2} \frac{GM\Delta m}{R_*}$ ⇒ luminosity lost during accretion is

$L_{disc} = \frac{1}{2} \frac{GM\dot{M}}{R_*}$, But grav pot energy is $\frac{GM\Delta m}{R_*}$, so 1/2 is radiated or dissipated in disk, other 1/2 released at surface of star

→ Compare fraction to ∡ momentum: $R^2\Omega(R) \propto R^{1/2}$

now since $R_{circ} \gg R_*$ in general nearly all ∡ momentum must be lost. → dissipation processes which cause conversion of kinetic energy to heat must also transport ∡ momentum

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